

VALIDITY OF THE GENERALIZED SECOND LAW OF THERMODYNAMICS OF THE UNIVERSE BOUNDED BY THE EVENT HORIZON IN HOLOGRAPHIC DARK ENERGY MODEL

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In this letter, we investigate the validity of the generalized second law of thermodynamics of the universe bounded by the event horizon in the holographic dark energy model. The universe is chosen to be homogeneous and isotropic and the validity of the first law has been assumed here. The matter in the universe is taken in the form of non-interacting two fluid system- one component is the holographic dark energy model and the other component is in the form of dust.

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I. INTRODUCTION

At present it is strongly believed that our universe is experiencing an accelerated expansion. The numerous cosmological observations suggest that the acceleration is driven by a missing energy density with negative pressure is called as dark energy. An approach to the problem of dark energy is holographic model[1-13]. The holographic principle states that the no. of degrees of freedom for a system within a finite region should be finite and is bounded roughly by the area of its boundary. From the effective quantum field theory one obtains the holographic energy density as[14]

$$\rho_D = 3c^2 M_p^2 L^{-2}$$

where L is an IR cut-off in units $M_p^2 = 1$. Li shows that[1] if we choose L as the radius of the future event horizon, we can get the correct equation of state and get the desired accelerating universe. Also in equation (1) c is any free dimensionless parameter whose value is determined by observational data[8,15-19]. However, in the present work we have taken c to be arbitrary.

From the string theory point of view the dimension of space-time should be more than four. However, Einstein-Hilbert(EH) action is the most general one in 4-D. For higher dimension general lagrangian contains Lovelock terms in addition to EH term and we have second order field equations. The simplest extension is to take the Gauss-Bonnet(GB)[19-24] term in addition to EH term. Further GB term is important both from geometrical and physical point of view. The GB term in the lagrangian is the higher curvature correction to general relativity and naturally arises as the next leading order of the α expansion of heterotic super string theory[25], where α^{-1} measures the string tension. Geometrically in five dimensions, Einstein-Hilbert action with GB term gives the most general lagrangian[26] for the second order field equations, while in four dimensions, the lagrangian corresponding to EH action is the most general one and GB term is only the topological one, which does not affect the dynamics[27].

In this work we have examined the validity of the generalized second law of thermodynamics of the universe bounded by the event horizon assuming the first law both for Einstein gravity as well as Gauss-Bonnet gravity.

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II. VALIDITY OF THE GENERALIZED SECOND LAW:

A. Einstein Gravity

In homogeneous and isotropic FRW model, the space-time metric can be written as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_2^2 \quad (1)$$

where $d\Omega_2^2 = d\theta^2 + (\sin^2\theta) d\phi^2$, is the metric on unit 2- sphere, $\tilde{r} = ar$ is the area radius and $h_{ab} = \text{diag}(-1, \frac{a^2}{(1-kr^2)})$ with $k = 0, \pm 1$ for flat, closed and open model. The Friedmann equations are (choosing $8\pi G = 1$)

$$H^2 + \frac{k}{a^2} = \frac{\rho_t}{3} \quad (2)$$

and

$$\dot{H} - \frac{k}{a^2} = -\frac{1}{2}(\rho_t + p_D) \quad (3)$$

with $\rho_t = \rho_m + \rho_D$. Here ρ_D and p_D correspond to energy density and thermodynamic pressure of the holographic dark energy model while ρ_m is the energy density corresponding to dust. As the two component matter system is non interacting so they satisfy the energy conservation equation separately, i.e.

$$\dot{\rho}_m + 3H(\rho_m) = 0 \quad (4)$$

and

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0 \quad (5)$$

We shall use the holographic dark energy (DE) model in reference [1]. As the universe is bounded by the event horizon so the energy density of the holographic model can be written as

$$\rho_D = 3c^2 R_E^{-2} \quad (6)$$

From above using the definition of the cosmological event horizon

$$R_E = a \int_a^\infty \frac{da}{Ha^2} = \frac{c}{(\sqrt{\Omega_D})H} \quad (7)$$

where $\Omega_D = \frac{\rho_D}{3H^2}$ is the density parameter corresponding to dark energy. The equation of state of the dark energy can be written as

$$\rho_D = \omega_D p_D \quad (8)$$

where ω_D is not necessarily a constant.

Now the amount of energy crossing the event horizon in time dt has the expression [28,29]

$$-dE = 4\pi R_E^3 H(\rho_t + p_D)dt \quad (9)$$

So assuming the validity of the first law of thermodynamics we have

$$\frac{dS_E}{dt} = \frac{4\pi R_E^3 H}{T_E} (\rho_t + p_D) \quad (10)$$

where S_E and T_E are the entropy and temperature of the event horizon respectively.

In order to obtain the variation of the entropy of the fluid (mentioned earlier) inside the event horizon we use the Gibb's equation [30,31]

$$T_E dS_I = dE_I + p_D dV \quad (11)$$

where S_I and E_I are the entropy and energy of the matter distribution and for the thermodynamical equilibrium the temperature of the matter distribution is assumed to be same as event horizon [32-34]. So starting with

$$E_I = \frac{4}{3}\pi R_E^3 \rho_t \quad \text{and} \quad V = \frac{4}{3}\pi R_E^3,$$

and using the Friedmann equations the Gibb's equation leads to

$$dS_I = \frac{4\pi R_E^2}{T_E} (\rho_t + p_D) dR_E + \frac{H R_E^3}{T_E} \left(\dot{H} - \frac{k}{a^2} \right) dt \quad (12)$$

Now to obtain the change in the radius of the event horizon (R_E) we start with the expression from holographic dark energy (i.e. equation (7)) and again using the Friedmann equations and the conservation equation (5) we get (after simplification)

$$dR_E = \frac{3}{2} R_E H (1 + \omega_D) dt \quad (13)$$

Hence using this expression of dR_E in (12) we get

$$\frac{dS_I}{dt} = \frac{2\pi R_E^3}{T_E} H (\rho_t + p_D) (3\omega_D + 1) \quad (14)$$

Hence combining (10) and (14) the resulting change of total entropy is given by

$$\frac{d}{dt}(S_I + S_E) = \frac{6\pi R_E^3 H}{T_E} (\rho_t + p_D) (\omega_D + 1) \quad (15)$$

or equivalently using the deceleration parameter $q = -1 - \frac{\dot{H}}{H^2}$ we obtain

$$\frac{d}{dt}(S_I + S_E) = \frac{12\pi R_E^3 H}{T_E} \left[(1 + q) H^2 + \frac{k}{a^2} \right] (\omega_D + 1) \quad (16)$$

The above results lead to the following conclusions :

I. The generalized second law of thermodynamics will be valid on the event horizon if the holographic dark energy component individually satisfies the weak energy condition i.e.

$$\rho_D + p_D = \rho_D (1 + \omega_D) > 0.$$

i.e if the holographic dark energy component is not of the phantom nature then universe as a thermodynamical system with two non-interacting fluid components (as in the present case) always obey the second law of thermodynamics. Also from equation (16) the expression within square bracket will be positive definite if $k = 0, +1$. For $k = -1$ we must have the inequality

$$\frac{1}{a^2 H^2} < (1 + q)$$

for the validity of the generalized second law of thermodynamics.

Further one may note that, in this case the entropy of the event horizon also increases with time while variation of the matter entropy with time is not positive definite, but the sum of the entropies increases with the evolution of the universe. Also the radius of the event horizon increases with time.

II. If we consider the two fluid system as a single fluid with energy density and pressure

$$\rho_t = \rho_m + \rho_D, \quad p_t = p_D$$

and assume the weak energy condition for the combined matter system then

$$\rho_t + p_t > 0$$

i.e.

$$\rho_m + \rho_D(1 + \omega_D) > 0.$$

Clearly the above inequality does not guarantee

$$(1 + \omega_D) > 0$$

i.e the weak energy condition for the holographic dark energy. Consequently from equations (15) and (16), the validity of the second law of thermodynamics is not definite. More over from equation(16) the expression within square bracket may be positive even if q is negative i.e. if

$$q > -1 - \frac{k}{a^2 H^2}.$$

Therefore the generalized second law may be valid both for a accelerating and decelerating phase of the universe.

B. Gauss-Bonnet Gravity:

Due to complicated form of the field equations we consider only the flat FRW model in GB theory. In $(n+1)$ -dimensional flat FRW model the modified Einstein equations in GB gravity are [29] (choosing $\frac{16\pi G}{n-1} = 1$)

$$H^2(1 + \tilde{\alpha}H^2) = \frac{\rho_t}{n} \quad (17)$$

and

$$\dot{H}(1 + 2\tilde{\alpha}H^2) = -\frac{\rho_t + p_D}{2} \quad (18)$$

with the conservation equations

$$\dot{\rho}_m + nH(\rho_m) = 0 \quad (19)$$

and

$$\dot{\rho}_D + nH(\rho_D + p_D) = 0 \quad (20)$$

As, before we have the same expression for the radius of the event horizon considering the holographic dark energy model [1] i.e.

$$R_E = \frac{c}{(\sqrt{\Omega_D})H}$$

where in GB theory density parameter has the expression

$$\Omega_D = \frac{\rho_D}{nH^2(1 + \tilde{\alpha}H^2)} \quad (21)$$

Thus using the above modified Einstein field equation a small variation of the radius of the event horizon is given by

$$dR_E = HR_E \left[\frac{n}{2}(1 + \omega_D) + \frac{\tilde{\alpha}\dot{H}}{(1 + \tilde{\alpha}H^2)} \right] dt \quad (22)$$

As the amount of energy crossing the horizon in time dt does not depend on any particular gravity theory so from the first law of thermodynamics the time variation of the entropy of the event horizon is given by

$$\frac{dS_E}{dt} = \frac{n\Omega_n R_E^n H}{T_E} (\rho_t + p_D) \quad (23)$$

where $\Omega_n = \frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)}$ is the volume of an n -dimensional unit ball.

Again using Gibb's law (11) and the variation of R_E (given by equation (21)) and proceeding as in Einstein gravity the rate of change of entropy (with respect to time) of the matter bounded by the event horizon can be expressed as

$$\frac{dS_I}{dt} = \frac{n\Omega_n R_E^n H}{T_E} (\rho_t + p_D) \left[\frac{n}{2}(1 + \omega_D) + \frac{\tilde{\alpha}\dot{H}}{(1 + \tilde{\alpha}H^2)} - 1 \right] \quad (24)$$

where we have to use the modified Einstein field equations (17)-(18) and the conservation equation (19), to derive the above expression.

Thus combining (22) and (23) the time variation of the total entropy is given by

$$\frac{d}{dt}(S_I + S_E) = \frac{n\Omega_n R_E^n H (\rho_t + p_D)}{2T_E} \left[n(1 + \omega_D) + \frac{2\tilde{\alpha}\dot{H}}{(1 + \tilde{\alpha}H^2)} \right] \quad (25)$$

or equivalently using deceleration parameter

$$\frac{d}{dt}(S_I + S_E) = \frac{n\Omega_n R_E^n H^3}{T_E} (1 + q)(1 + 2\tilde{\alpha}H^2) \left[n(1 + \omega_D) - \frac{2\tilde{\alpha}(1 + q)H^2}{(1 + \tilde{\alpha}H^2)} \right] \quad (26)$$

In this case we cannot make any definite conclusion regarding validity of the second law of thermodynamics. The term within the square bracket in the r.h.s. of the equation (24) (or (25)) suggests that the equation of state for the holographic dark energy should be restricted by a complicated expression involving H and \dot{H} . Finally note that, as $\alpha \rightarrow 0$ we get back our earlier results in Einstein gravity.

III. CONCLUSIONS:

In the present work we examine the validity of the generalized second law of thermodynamics on the event horizon assuming the validity of the first law of thermodynamics both in Einstein gravity as well as in Gauss-Bonnet gravity. We consider the universe as a thermodynamical system and is filled with non-interacting two fluids. Here one component has been considered as the holographic dark energy and other is in the form of dust. One may note that we have not used the blackhole entropy as the entropy of the event horizon. In the first section we have shown that if the weak energy condition is satisfied by the holographic dark energy component

on the event horizon then the generalized second law is valid there provided the first law is valid on the event horizon. It is to be noted that a similar study was done by Horvat[35] for universe filled with holographic dark energy. So our work may be considered as a generalization of his work with non-interacting two fluid system. The next section deals with the flat FRW model in Gauss-Bonnet theory. Here also considering the holographic dark energy scenario in the frame work of Gauss-Bonnet theory we have studied the generalized second law of thermodynamics on the event horizon assuming the validity of the first law. But due to complicated form of the expressions in Gauss Bonnet gravity we can not make any definite conclusion as in the previous case. The criteria of the validity of the first law of thermodynamics on the event horizon will be interesting and subsequently we shall study it.

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References:

- [1] M. Li , *Phys. Lett. B* **603** 01 (2004);
- [2] M. R. Setare and S. Shafei , *JCAP* **0609** 011 (2006) arXiv: gr-qc/0606103 ;
- [3] B. Hu and Y. Ling *Phys. Rev. D* **73** 123510 (2006).
- [4] B. Wang , Y. Gong , E. Abdalla, *Phys. Lett. B* **624** 141 (2005).
- [5] M. Ito , *Europhys. Lett.* **71** 712 (2005);
- [6] S. Nojiri , S. Odintsov , *Gen. Rel. Grav.* **38** 1285 (2006);
- [7] E.N. Saridakis , *Phys. Lett. B* **660** 138 (2008);
- [8] Q.G. Huang and M. Li , *JCAP* **0408** 013 (2004);
- [9] X. Zhang , *IJMPD* **14** 1597 (2005);
- [10] D. Pavon and W. Zimdahl , *Phys. Lett. B* **628** 206 (2005);
- [11] H. kim , H.W. Lee and Y.S. Myung , *Phys. Lett. B* **632** 605 (2006);
- [12] S.D. Hsu , *Phys. Lett. B* **594** 01 (2004);
- [13] R. Horvat , *Phys. Rev. D* **70** 087301 (2004);
- [14] A.G. Cohen , D.B. Kaplan and A.E. Nelson , *Phys. Rev. Lett.* **82** 4971 (1999);
- [15] Z. Chang, F-Q. Wu and X. Zhang, *Phys. Lett. B* 633, 14 (2006) [arXiv:astro-ph/ 0509531].
- [16] H. C. Kao, W. L. Lee and F. L. Lin, astro-ph/0501487; X. Zhang and F-Q. Wu, *Phys. Rev. D* 72, 043524 (2005) [arXiv:astro-ph/0506310]; X. Zhang and F-Q. Wu, *Phys. Rev. D* 76, 023502 (2007) [arXiv:astro-ph/0701405].
- [17] Q. Wu, Y. Gong, A. Wang and J. S. Alcaniz, [arXiv:astro-ph/0705.1006]; Y-Z. Ma and Y. Gong, [arXiv:astro-ph/0711.1641].
- [18] J. Shen, B. Wang, E. Abdalla and R. K. Su, [arXiv:hep-th/0412227].
- [19] E.N. Saridakis and M.R. Setare , *Phys. Lett. B* **670** 01 (2008);

- [20] *E.N. Saridakis* , *Phys. Lett. B* **661** 335 (2008);
- [21] *C. Charmouis and J.F. Dufaux* , *CQG* **19** 4671 (2002);
- [22] *J.E. Kim* , *B. Kyae and H.M. Lee* , *Nucl. Phys. B* **582** 296 (2000);
- [23] *E. Gravanis and S. Willison* , *Phys. Lett. B* **562** 118 (2003);
- [24] *R.G. Cai* , *H.S. Zhang and A. Wang* , *Commun Theor. Phys.* **44** 948 (2005);
- [25] *P. Candelas* , *G.T. Horowitz* , *A. Strominger and E. Witten* *Nucl. Phys. B* **258** 46 (1985);
M.B. Greens, *J.H. Schwarz and E. Witten* ,”*Super String Theory*”, *Cambridge University Press* (1987); *J. Polchinski* , ”*String Theory*”, *Cambridge University Press, Cambridge*, (1998).
- [26] *C. Lanczos* ,*Ann. Math* **39** 842 (1938).
- [27] *M. Sami and N. Dadhich* , *TSPU Vestnik* **44N7** 25 (2004) (*arXiv: hep-th /0405016*).
- [28] *R.S. Bousso*, *Phys. Rev. D* **71** 064024 (2005).
- [29] *R. G. Cai and S. P. Kim*, *JHEP* **02** 050 (2005).
- [30] *G. Izquierdo and D. Pavon*, *Phys. Lett. B* **633** 420 (2006).
- [31] *B. Wang, Y. Gong, E. Abdalla*, *Phys. Rev. D* **74** 083520 (2006).
- [32] *E N. Saridakis, P.F. Gonzalez-Daz and C.L. Siguenza* , *arXiv: 0901.1213/astro-ph*;
- [33] *P.F. Gonzalez-Daz and C.L. Siguenza* ,*Nucl. Phys. B* **697** 363 (2004);
- [34] *S. H. Pereira and J.A.S. Lima* , *Phys. Lett. B* **669** 266 (2008);
- [35] *R. Horvat* , *Phys. Lett. B* **664** 201 (2008);